AIMS AND OBJECTIVES

The aim of the Faculty of Mathematics is to provide relevant service courses for the Natural Sciences and Computer Science Triposes. After completing Mathematics A or B (Part IA) and Mathematics (Part IB), students should have covered the mathematical methods required to provide a grounding in the mathematical techniques used either in the Physical Sciences courses of the Natural Sciences Tripos or in the Computer Science Tripos, as appropriate.

COURSES

Part IA

The following mathematics courses are provided for Part IA of the Natural Sciences and Computer Science Tripos.

- Mathematics, Course A
- Mathematics, Course B

Course A provides a thorough grounding in methods of mathematical science and contains everything prerequisite for the mathematical content of all physical-science courses in Part IB of the Natural Sciences Tripos, including specifically Mathematics, Physics (Physics A) and Advanced Physics (Physics B). Course B contains additional material for those students who find mathematics rewarding in its own right, and it proceeds at a significantly faster pace. Both courses draw on examples from the physical sciences but provide a general mathematical framework by which quantitative ideas can be transferred across disciplines.

Students are strongly encouraged to take Course A unless they have a thorough understanding of material in Further Mathematics A-Level. As a guide, such students might be expected to have scored in the region of 95% in at least two of the modules FP1, FP2, FP3. Some topics that look similar in the Schedules may be lectured quite differently in terms of style and depth. Both courses lead to the same examination and qualification. Mathematics is a skill that requires firm foundations: it is a better preparation for future courses in NST to gain a first-class result having pursued Course A than to gain a second-class result following Course B.

Each course consists of 60 lectures over three terms.

Part IA: Mathematics, course B

This course comprises Mathematical Methods I, Mathematical Methods II and Mathematical Methods III.

The material will be as well illustrated as time allows with examples and applications of Mathematical Methods to the Physical Sciences.

Mathematical Methods I — 24 lectures, Michaelmas term

Vector sum and vector equation of a line. Scalar product, unit vectors, vector equation of a plane. Vector product, vector area, vector and scalar triple products. Orthogonal bases. Cartesian components. Spherical and cylindrical polar coordinates. [**4**]

Complex numbers and complex plane, vector diagrams. Exponential function of a complex variable. $\exp(\mathrm{i}\omega t)$, complex representations of cos and sin. Hyperbolic functions. [**2**]

Revision of single variable calculus. Leibnitz's formula. Elementary curve sketching. Elementary Analysis; idea of convergence and limits. Orders of magnitude and approximate behaviour for large and small x. O notation. Idea of continuity and differentiability of functions. Power series. Statement of Taylor's theorem. Examples to include binomial expansion, exponential and trigonometric functions, and logarithm. Newton-Raphson method. Convergence of series; comparison and ratio tests. [**6**]

The integral as a sum, differentiation of an integral with respect to its limits or a parameter. Approximation of a sum by an integral. Stirling's approximation as an example. Schwarz's inequality. Double and triple integrals in Cartesian, spherical and cylindrical coordinates. Examples to include evaluation of $\int_{-\infty}^{+\infty} \exp(-x^2)\,\mathrm{d}x$. [**5**]

Elementary probability theory. Simple examples of conditional probability. Probability distributions, discrete and continuous, normalisation. Permutations and combinations. Binomial distribution, $(p+q)^n$, binomial coefficients. Normal distribution. Expectation values, mean, variance, $\sigma^2 = < x^2 > - < x >^2$. [**4**]

Extended examples distributed through the course. [**3**]

Mathematical Methods II **24 lectures, Lent term**

Ordinary differential equations. First order equations: separable equations; linear equations, integrating factors. Examples involving substitution. Second-order linear equations with constant coefficients; $\exp(\lambda x)$ as trial solution, including degenerate case. Superposition. Particular integrals and complementary functions. Constants of integration and number of necessary boundary/initial conditions. Particular integrals by trial solutions. Examples including radioactive sequences. Resonance, transients and damping. [**6**]

Differentiation of functions of several variables. Differentials, chain rule. Exact differentials, illustrations including Maxwell's relations. Scalar and vector fields. Gradient of a scalar as a vector field. Directional derivatives. Unconditional stationary values; classification using Hessian matrix. Conditional stationary values, Lagrange multipliers, examples with two or three variables. Boltzmann distribution as an example. [**8**]

Line integral of a vector field. Conservative and non-conservative vector fields. Surface integrals and flux of a vector field over a surface. Divergence of a vector field. ∇^2 as div grad. Curl. Divergence and Stokes's theorems. [**5**]

Orthogonality relations for sine and cosine. Fourier series; examples. [**2**]

Extended examples distributed through the course. [**3**]

Mathematical Methods III **12 lectures, Easter term**

Linear equations. Notion of a vector space; linear mappings. Matrix addition and multiplication. Determinant of a matrix. Statement of the main properties of determinants. Inverse matrix. Equations $\mathbf{A}\,\mathbf{x} = \mathbf{0}$ with non-zero solutions. Symmetric, antisymmetric and orthogonal matrices. Eigenvalues and eigenvectors for symmetric matrices. [**6**]

Linear second-order partial differential equations; physical examples of occurrence, verification of solution by substitution. Linear superposition. Method of separation of variables (Cartesian coordinates only). [**4**]

Extended examples distributed through the course. [**2**]

BIBLIOGRAPHY

There are very many books which cover the sort of mathematics required by Natural Scientists. The following should be helpful as general reference; further advice will be given by Lecturers. Books which can reasonably be used as principal texts for the course are marked with a dagger.

Natural Sciences Mathematics Part IA

† M L Boas
Mathematical Methods in the Physical Sciences, 2nd edition.
Wiley, 1983 (£37.95 hardback)
(3rd edition available August 2005, £34.95 hardback).

A Jeffrey
Mathematics for Engineers and Scientists, 5th edition.
Nelson Thornes, 1996 (£26.32 paperback)
(6th edition available, Blackwells, £29.99)

† E Kreyszig
Advanced Engineering Mathematics, 8th edition.
Wiley, 1999 (£34.95 paperback, £83.50 hardback)
(9th edition available, £34.95 hardback).

† K F Riley, M P Hobson & S J Bence
Mathematical Methods for Physics and Engineering.
2nd ed., Cambridge University Press, 2002 (£33.00 paperback).

I S Sokolnikoff & R M Redheffer
Mathematics of Physics and Modern Engineering, 2nd edition.
McGraw Hill, 1967 (out of print)

† G Stephenson
Mathematical Methods for Science Students, 2nd edition.
Prentice Hall/Pearson, 1973 (£35.99 paperback).

G Stephenson
Worked Examples in Mathematics for Scientists and Engineers.
Longman, 1985 (out of print)

K A Stroud & D Booth
Engineering Mathematics, 5th edition.
Palgrave, 2001 (£30.99 paperback with CD-ROM)

K A Stroud & D Booth
Advanced Engineering Mathematics.
Palgrave, 2003 (£32.99 paperback)

G Thomas, M Weir, J Hass & F Giordano
Thomas's Calculus, 11th edition.
Pearson, 2004 (£45.99 hardback)