Part IB: Mathematics

This course comprises Mathematical Methods I, Mathematical Methods II, Mathematical Methods III and six Computer Practicals. The material in Course A from Part IA will be assumed in the lectures for this course. Topics marked with asterisks should be lectured, but questions will not be set on them in examinations.

The material in the course will be as well illustrated as time allows with examples and applications of Mathematical Methods to the Physical Sciences.

Mathematical Methods I — 24 lectures, Michaelmas term

Vector calculus
Suffix notation. Contractions using δ_{ij} and ϵ_{ijk}. Reminder of vector products, grad, div, curl, ∇^2, and their representations using suffix notation. Divergence theorem and Stokes' theorem. Vector differential operators in orthogonal curvilinear coordinates, e.g. cylindrical and spherical polar coordinates. Jacobians. [**6**]

Partial differential equations
Linear second-order partial differential equations; physical examples of occurrence, the method of separation of variables (Cartesian coordinates only). [**2**]

Green's functions
Response to impulses, delta function (treated heuristically), Green's functions for initial and boundary value problems. [**3**]

Fourier transform
Fourier transforms; relation to Fourier series, simple properties and examples, convolution theorem, correlation functions, Parseval's theorem and power spectra. [**2**]

Matrices
N–dimensional vector spaces, matrices, scalar product, transformation of basis vectors. Eigenvalues and eigenvectors of a matrix; degenerate case, stationary property of eigenvalues. Orthogonal and unitary transformations. Quadratic and Hermitian forms, quadric surfaces. [**5**]

Elementary Analysis
Idea of convergence and limits. O notation. Statement of Taylor's theorem with discussion of remainder. Convergence of series; comparison and ratio tests. Power series of a complex variable; circle of convergence. Analytic functions: Cauchy-Riemann equations, rational functions and $\exp(z)$. Zeros, poles and essential singularities. [**3**]

Series solutions of ordinary differential equations
Homogeneous equations; solution by series (without full discussion of logarithmic singularities), exemplified by Legendre's equation. Classification of singular points. Indicial equation and local behaviour of solutions near singular points. [**3**]

Computer practicals **Michaelmas & Lent terms**

There are no lectures for this course, which consists of six computational exercises related to material elsewhere in the Mathematics course.

Topics for the exercises will include :

1. Familiarisation, getting started. Numerical integration.
2. Solving ordinary differential equations.
3. Root finding.
4. Solving partial differential equations.
5. Matrix algebra.
6. Eigenfunction expansions.

Mathematical Methods II **24 lectures, Lent term**

Sturm-Liouville theory
Self-adjoint operators, eigenfunctions and eigenvalues, reality of eigenvalues and orthogonality of eigenfunctions. Eigenfunction expansions and determination of coefficients. Legendre polynomials; orthogonality. [**3**]

Conditional stationary values and the calculus of variations
Lagrange multipliers, examples with two or three variables. Euler-Lagrange equations and examples.

Variational principles; Fermat's principle; Hamilton's principle and deduction of Lagrange's equation, illustrated by a system with:

$$L = \tfrac{1}{2}\, m_1 \dot{x}_1^2 + \tfrac{1}{2}\, m_2 \dot{x}_2^2 - V(x_1 - x_2)\,.$$

Variational principle for the lowest eigenvalue *and for higher eigenvalues* (Rayleigh-Ritz). Eigenvalues of perturbed operators. [**6**]

Laplace and Poisson's equations
Solution by separation of variables of Laplace's equation in plane polar coordinates, and spherical polar coordinates (axisymmetric case); Legendre polynomials again.

Solution of Poisson's equation as an integral. Uniqueness for Poisson's equation with Dirichlet boundary conditions. Green's identity. Green's function for Laplace's equation with simple boundary conditions using the method of images. Applications to electrostatic fields and steady heat flow. [**5**]

Cartesian tensors
Transformation laws, addition, multiplication, contraction. Isotropic tensors, symmetric and anti-symmetric tensors. Principal axes and diagonalisation. Tensor fields, e.g. conductivity, polarizability, elasticity. [**4**]

Contour integration
Integration along a path; elementary properties. Cauchy's theorem; proof by Cauchy-Riemann equations and divergence theorem in 2–D. Integral of $f'(z)$; Cauchy's formula for $f(z)$. Calculus of residues; examples of contour integration; point at infinity; multi-valued functions, branch points, $\log\,(z)$. [**4**]

Transform methods
Fourier inversion by contour integration. Examples of simple linear differential equations, including diffusion equation. [**2**]

Mathematical Methods III **10 lectures, Easter term**

Small oscillations

Small oscillations and equilibrium; normal modes, normal coordinates, examples, e.g. vibrations of linear molecules such as CO_2. Symmetries of normal modes. [**2**]

Group theory

Idea of an algebra of symmetry operations; symmetry operations on a square. Definition of a group; group table. Subgroups; homomorphic and isomorphic groups.

Representation of groups; reducible and irreducible representations; basic theorems of representation theory. Classes, characters. Examples of character tables of point groups. *Applications in Molecular Physics*. [**8**]

Natural Sciences Mathematics Part IB

† G Arfken & H Weber
Mathematical Methods for Physicists, 6th edition.
Elsevier, 2005 (£59.99 hardback).

† J W Dettman
Mathematical Methods in Physics and Engineering.
Dover, 1988 (£12.95 paperback).

† H F Jones
Groups, Representation and Physics, 2nd edition.
Institute of Physics Publishing (Taylor & Francis), 1998 (£24.99 paperback).

E Kreyszig
Advanced Engineering Mathematics, 8th edition.
Wiley, 1999 (£34.95 paperback, £83.50 hardback)
(9th edition available, £34.95 hardback)

J Mathews & R L Walker
Mathematical Methods of Physics, 2nd edition.
Pearson/Benjamin Cummings, 1970 (£68.99 paperback)

† K F Riley, M P Hobson & S J Bence
Mathematical Methods for Physics and Engineering.
2nd ed., Cambridge University Press, 2002 (£33.00 paperback).

R N Snieder
A guided tour of mathematical methods for the physical sciences, 2nd edition.
Cambridge University Press, 2004 (£30.00 paperback)